



Establishing Equivariant Class $[O]$ for Hyperbolic Groups

Deep Bhattacharjee ^{a*}

^a Theoretical Physics Research Division of AATWRI Aerospace and Defense Research Directorate, Electro Gravitational Space Propulsion Laboratory, India.

Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/ARJOM/2022/v18i11615

Open Peer Review History:

This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: <https://www.sdiarticle5.com/review-history/92728>

Received 22 August 2022

Accepted 25 October 2022

Published 31 October 2022

Original Research Article

Abstract

This paper aims to create a class $[O]$ concerning the groups associated with Gromov hyperbolic groups over correspondence and equivalence through Fuchsian, Kleinian, and Schottky when subject to Laplace – Beltrami in the Teichmüller space where for the hyperbolic 3-manifold when the fundamental groups of Dehn extended to Gromov – any occurrence of Švarc-Milnor lemma satisfies the same class $[O]$ for quotient space and Jørgensen inequality. Thus the relation (and class) extended to Mostow – Prasad Rigidity Theorem in a finite degree isometry concerning the Quasi – Isomorphic structure of the commensurator in higher order generalizations suffice through $CAT(k)$ space. The map of the established class $[O]$ is shown at the end of the paper.

Keywords: Teichmüller Space; Dehn; Švarc-Milnor; Jørgensen Inequality; Laplace – Beltrami; Lickorish – Wallace; Haken Space.

1 Introduction

Any non-Euclidean geometry having a saddle or negative curvature where both the omega and Kretschmann scaler is less than 1 being defined for Riemann curvature tensor R_{mnop} there lies a vanishing Ricci R_{mn} through [1],

$$K = R_{mnop}R^{mnop} \text{ for } deg[L]^{-4}$$

*Corresponding author: Email: deep.bh@outlook.com;

Every Gauss-Bolyai-Lobachevsky Space is a Riemann space provided it is a symmetric non-compact type where the Gaussian Curvature G being negative gives the inverse root where there exists a limit in the Geodesic curvature for 2-such cases as identified [2],

$$\frac{1}{\sqrt{-G}} = \begin{cases} 1, & \text{Horocycle} \\ [0,1], & \text{Hypercycle} \end{cases}$$

Riemann and Hyperbolic space are equivalent over actions on PGL groups for any isomorphism over 3-manifold as the Dehn surgery segregates the Flat (Euclidean) from hyperbolic provided any 3-manifold can take on different geometries for the same structures where the conformal boundary associated with the Riemann sphere can be distinguished by group [3],

$$\text{PGL}(2, \mathbf{C})$$

2 Gauss-Bolyai-Lobachevsky Theorem with Dehn Surgery

Being associated with the homology classes and established over simplicial norm there exists the Dehn Link and Dehn Twists satisfying the properties [4],

- [1] *Lickorish – Wallace Theorem* – Where Dehn Twist D can be defined as an automorphism for roundabout channel C such that the mapping factor φ gives,

$$\varphi: C \rightarrow C \text{ over time } T \text{ giving } D \cong Ce^{i2\pi T} \exists T \in [0,1]$$

- [2] *Manifold Type* – Closed and orientable.

3 Fuchsian Group

By taking the Gauss-Bolyai-Lobachevsky over the associated and irreducible prescriptions of Dehn Twist the $\text{PGL}(2, \mathbf{C})$ preserves the isometry for any Hyperbolic 3-manifold – a category where the necessary functors of orientations suffice the Möbius Transform group we get a Kleinian group representing the Riemann sphere over conformal transformations in $\text{PSL}(2, \mathbf{C})$. Thus by considering 2 factors a realization can be made for connections of Fuchsian to Kleinian groups [5],

- [1] Categorizing Kleinian as the discrete subgroup ω and the hyperbolic 3-manifold as M_H^3 then two cases can be established over the equation,

- a. Co-compact being finitely co-volume
- b. Co-volume being finitely generated

- i. $Over - M_H^3/\omega$

- [2] For the real domain R of $\text{PSL}(2, \mathbf{C})$ then over any finite generations being an isometry group taking on the upper half of the unit disc throughout any conformal transformations – it can be shown that [6],

- a. M_H^3 is discontinuous over $\text{PSL}(2, \mathbf{R})$ when Kleinian takeover the Fuchsian for the discrete subgroup ω .
- b. For any polynomial P there exists a non-repetitive root for the equation

$$x^3 + ax + b$$

\exists there exists a deg_3 Potential of P in x for every y^2 establishing the above equation

- This again gives another subgroup ω^0 satisfying the moduli space of y^2 via $\text{PSL}(2, \mathbf{Z})$
- Fuchsian norms for non-abelian ω .

- $PSL(2, \mathbb{C})$ is then satisfied for the hyperbolic 3-fold over functors M_H^3 of Möbius group where a 2-sphere ${}_B^3S^2$ is taken at infinity being associated with the conformal homeomorphism where there exists an associated hyperbolic isometry on 3-ball B^3 . Thus the discontinuity as mentioned in [a] of point [2] gives an extension to 2 categories [7],
- A. Schottky group over any Hausdorff measure $h < 2$ for the discrete subgroup ω for $2g$ weight for a total convergence over the bundle norm $b \geq 1$ over linear fractional transformations $\partial \equiv \frac{\partial_1}{\partial_2}$,

$$\sum_{\partial \in \omega} \partial_2^{-2g} h \partial \exists \text{ as mentioned earlier } H \text{ is Hausdroff } < 2 \cong \text{Poincaré series } b \geq 1$$

- B. Denoting area of discontinuity D^0 we again get 2 relations,
 1. *Orbifold Riemann* – for the discrete subgroup ω there is $D^0(\omega)/\omega$
 2. *Fuchsian* \equiv *Schottky* – For area inequality D^0/ω .

4 Teichmüller Space

Any Teichmüller space $T(M)$ can be defined over a Riemann manifold M endowed with a hyperbolic structure where there exists identity homeomorphism. For any universal covering making an identification for genus $g \geq 2$ - The compact structure or the surface that the topology establishes prescribed 5 interrelated parameters [8],

- [1] Non-compact space
- [2] Teichmüller space $T(M)$
- [3] Fuchsian group
- [4] Isotopy norm over Riemann M for metric g_M suffice smoothness s ,

$$s \sum_{i=1}^n dx_i^2$$

- [5] Uniformization theorem for Hyperbolic 3-folds taking Thurston’s 8-geometries taking over [9],

- Point [4] noting $s \sum_{i=1}^n dx_i^2$ as metric $d\ell^2$ giving,
 1. Hodge star \star for isothermal coordinates equipped with above metric $d\ell^2$ such that for smoothness s there exists *Jacobean* $\neq 0$,
 2. s is equipped with the differential $\partial\alpha^2 + \partial\beta^2$ (Let α, β be the coordinates where \star generates 3 peculiar forms,
 - $\star \partial \star \partial$
 - $\star \partial\alpha = \partial\beta$
 - $\star \partial\beta = \partial\alpha$
- *Laplace – Beltrami* – For the exterior derivative (being non-trivially hidden in sub-points 1 and 2 under [5]) we would be getting a scalar potential ϕ such that this can be differentiated for the Riemann metric g_M over [10],

$$\partial_m |g_M|^{-1/2} (\partial_m \phi g_M^{mn} |g_M|)$$

Giving area inequality D^0/ω for discrete subgroup ω with a discontinuity $D^0 - Schottky \cong Fuchsian$ provided Poincaré series stands at $b \geq 1$. While all this occupying genus g Teichmüller is identified with the Fuchsian over 3 norms [11],

1. Genus $g \geq 2$
2. Ball B having deg_{6g-6}
3. For scalar potential ϕ with Riemann $\partial_m |g_M|^{-1/2} (\partial_m \phi g_M^{mn} |g_M|)$ there exists [12],
 - a. $T(M) \xrightarrow{\text{bijection}} (M, \phi)$
 - b. Equivalence class $[O]$ for the isotopy, diffeomorphism, and holomorphism such that for closed interval $[0,1]$ Teichmüller gives [13],
 1. $T(M) \Rightarrow (g_M, M)$
 2. Diffeomorphism to ϕ for contractable $\mathbb{R}^2 \forall 6g - 6$ making an equivalence as,

$$[O] \equiv T(M) \cong \mathbb{R}^2 \forall 6g - 6 \exists Fuchsian \equiv Schottky \equiv Kleinian \Rightarrow \text{Lie Group } \text{PSL}_2(\mathbb{R})$$

5 Gromov Hyperbolic Groups

No 3-dimensional manifold can occupy a single class of geometry. Where either there exists a certain curvature on the outside while a certain curvature is inside. In some cases – the structure can be equipped with the surface geometry taking different curvatures on different parts to say the frequency concerned with that manifold. There have been groups equipped with structures for the homotopy invariant spaces but the segregation can be possible by the Dehn surgery for making a distinct classification from Euclidean to Hyperbolic geometries. Any isomorphisms can be satisfied by this homotopy invariant spaces M_X that are Riemann with a negative curvature. Thus taking X as the topological space for the negative Riemann M - one can deduce the fundamental groups for a completely connected path along the surface being parameterized by P along X [14],

$$\rho_1(X_P)$$

If we take another topological space Y for hyperbolic manifold M_Y then we can deduce 3 relations making a symmetric equivalence for class $[O]$ [15],

1. For any hyperbolic 3-manifold $\rho_1(X_P)$ satisfies a quasi-isometric form such that for any definite deg_{finite} Lie Group \mathcal{L} one can find the Riemann metric g_M where there exists closed-connectedness between the topological manifold M_X or M_Y satisfying quotients via [16],

$$\check{S}varc - \text{Milnor lemma} = \left\{ \begin{array}{l} X/\mathcal{L} \\ Y/\mathcal{L} \end{array} \forall \rho_1(X_P) \simeq \rho_1(Y_P) \exists \wedge: M_X \rightarrow M_Y \right.$$

2. The quotient topology norm with *Švarc – Milnor lemma* is indeed equivalent to the same fundamental group denoted as $G_{\mathcal{L}}$ suffice [17],

$$\left\{ \begin{array}{l} \rho_1(X_P) \rightarrow \rho_1(X_P/G_{\mathcal{L}}) \\ \rho_1(Y_P) \rightarrow \rho_1(Y_P/G_{\mathcal{L}}) \end{array} \right. \cong G_{\mathcal{L}} \text{ Note 1}$$

3. For establishing equivariant hyperbolic groups for class $[O]$ – The hyperbolic 3-manifold M_H^3 suffices an inequality relation over the same class correspondence – $[O]$ – [18]

$$[O] \equiv \left\{ \begin{array}{l} \text{Quotient Topological Space } M_H^3/G_{\mathcal{L}} \\ \text{Jørgensen inequality} \end{array} \right. \cong \text{Kleinian Groups}$$

Note 1 – Higher order generalizations for the fundamental group can be achieved by making $\rho_n(X_P)$ and $\rho_n(Y_P)$ for closed-connectedness between the topological manifold M_X or $M_Y \exists \wedge: M_X \rightarrow M_Y$ where in the case of complex plane \mathbb{Z} – There are higher-order homotopy groups of n – spheres S^n in the ρ_n order making as $\rho_n(S^n)$

If we take any complex plane \mathbb{C} and suffice that any element existing on that complex plane gives the trace μ^1 and μ^1 of a 2×2 matrix then any parabolic function gives the cusp of a Riemann 3-fold that iff generates through the Kleinian Groups then – This suffice a correlation for any suitable parameters belonging to those traces μ^1 and μ^1 norms,

$$\cong \text{Fuchsian Group for } \mathbb{U}^{\frac{1}{2}}/G_{\mathcal{G}} \ni \gamma := \mathbb{U}^{\frac{1}{2}}/G_{\mathcal{G}} \text{ Note } 2$$

This again suffices the $[O]$ – class in a more concrete structure.

6 Mostow – Prasad Rigidity Theorem

If we take the same fundamental group of Dehn which Gromov extended – for the hyperbolic 3-fold then we will again find ourselves embedded in the Teichmüller space for every deg_{finite} closed manifolds one would surprisingly find the $[O]$ – class equivariant as stated in [Page 5] for manifold M suffice Teichmüller $T(M)$ in contractable \mathbb{R}^2 for all that norm $6g - 6$ taking the Lie Group $PSL_2(\mathbb{R})$ [19].

For assessment of clarifications when the connected path P along topological space X for the concerned manifold M_X – as we have seen in Švarc – Milnor lemma where the established equation of path connectivity $\rho_i(X_P)$ takes [20],

$$\sum_{i=1}^n \rho_i \bigvee_{p \in p^n} X_P$$

The map $\wedge: M_X \rightarrow M_Y$ goes prominent in the same deg_{finite} isometry where the fundamental group $\rho_i \ni i$ takes $[1, n]$ – the same map can be re-written through this group structure,

$$\wedge: \sum_{i=1}^n \rho_i(M_X) \rightarrow \sum_{i=1}^n \rho_i(M_Y)$$

The same class can be satisfied for n – sphere via any higher degree generalizations – where $M_H^3 \rightarrow M_H^n$ via,

$$\rho_n(S^n)$$

The large-scale structures are highlighted for any established deg_{finite} connectivity where a peculiar functor **Comm** takes over the generators of the group $G_{\mathcal{G}}$ where there exists the path-connectivity for some arbitrary commensurator Π over any intersections of sub-group $G_{\mathcal{G}}^1$ and $G_{\mathcal{G}}^2$ such that the **Comm** over group $G_{\mathcal{G}}$ establishes [21],

Comm(Π) justifies $G_{\mathcal{G}}^1, G_{\mathcal{G}}^2 \in G_{\mathcal{G}}$ iff $\Pi \equiv$ Quasi – Isomorphic

Hence a $[O]$ – class equivariant relation,

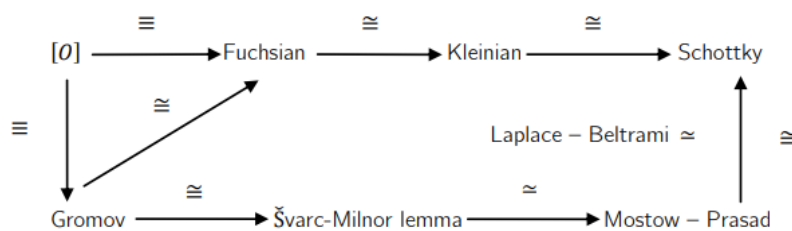


Fig. 1. This image depicts the equivariant class $[O]$ to establish the objective of this paper

Note 2 – $\mathbb{U}^{\frac{1}{2}}$ denotes the upper half-plane where for any projections parameterized by γ there exists a difference in the relational subset for Group – G_L from its mean $\langle G_L \rangle$

This indeed suffices for an argument if we consider the fundamental group considering n – sphere $\rho_n(S^n)$ where there exists a $CAT(k)$ space for every M_H^n for manifold M_X equipped with metric g such that this suffice for every $\leq k$ in $CAT(k)$ [22,23],

$$(M_X, g) \left\{ \begin{array}{l} M_H^n \left\{ \begin{array}{l} CAT(k) \text{ for } k = -1 \\ \cdot \\ \text{Hadamard space for } k = 0 \\ \cdot \\ \cdot \\ \text{Fundamental group } \rightarrow \rho_n(S^n) \\ \cdot \\ S^n \left\{ \begin{array}{l} \cdot \\ CAT(k) \text{ for } k = 1 \end{array} \right. \end{array} \right. \end{array} \right.$$

Thus we get $[O] \equiv T(M) \cong \mathbb{R}^2 \forall 6g - 6 \exists Fuchsian \equiv Schottky \equiv Klenia \implies \text{Lie Group } PSL_2(\mathbb{R})$ where there can be $CAT(k)$ space for every M_H^n as (M_X, g) with g being the equipped metric. ^{Note 3}

7 Results

Equivariance is satisfied for class $[O]$ with the necessary hyperbolic groups – that being considered for this paper. The extended relation that is shown is the large-scale and also higher-degree generalization when subject to specific Lemma and Theorem mentioned throughout this paper. Concerning the Gauss-Bolyai-Lobachevsky space, the equivariant being satisfies over equivalence among *Fuchsian* \equiv *Schottky* \equiv *Klenia* all being subject to Gromov and Thurston’s 8-geometries to suffice the related extension from M_H^3 to M_H^n all being justified through the fundamental group, $CAT(k)$ space and Hadamard space for the associated metric concerned with the hyperbolic manifold (M_X, g) .

8 Conclusion

It is concluded that the relation (and class) extended to Mostow – Prasad Rigidity Theorem in a finite degree isometry concerning the Quasi – Isomorphic structure of the commensurator in higher order generalizations suffice through $CAT(k)$ space.

Disclaimer

This paper is an extended version of a preprint document of the same author. The preprint document is available in this link: <https://easychair.org/publications/preprint/IJFD> [As per journal policy, pre-print article can be published as a journal article, provided it is not published in any other journal]

Competing Interests

Author has declared that no competing interests exist.

References

[1] Bhattacharjee D. Establishing equivalence among hypercomplex structures via Kodaira embedding theorem for non-singular quintic 3-fold having positively closed (1,1)-form Kähler potential $i 2^{-1} \partial \bar{\partial}^* \rho$. Research Square; 2022r. Available:<https://doi.org/10.21203/rs.3.rs-1635957/v1>

[2] Bolyai-Gauss-Lobachevsky C, Lovas I, Jenkovszky L. Conference Bolyai-Gauss-Lobachevsky. BGL-2. Akadémiai Kiadó; 1999.

Note 3 – The Lie Group will not be the same as $PSL_2(\mathbb{R})$ for casewise alterations.

- [3] Milne JS. Algebraic groups: The theory of group schemes of finite type over a field (Cambridge Studies in Advanced Mathematics, Series Number 170) (1st ed.). Cambridge University Press; 2017b.
- [4] Qiu R. Reducible dehn surgery and annular dehn surgery. Pacific Journal of Mathematics. 2000;192(2):357–368.
Available:<https://doi.org/10.2140/pjm.2000.192.357>
- [5] Coornaert M, Papadopoulos A. Symbolic dynamics and hyperbolic groups (Lecture Notes in Mathematics, 1539) (1993rd ed.). Springer; 1993.
- [6] Bhattacharjee D. An outlined tour of geometry and topology as perceived through physics and mathematics emphasizing geometrization, elliptization, uniformization, and projectivization for Thurston's 8-geometries covering Riemann over Teichmuller spaces. TechRxiv; 2022e.
Available:<https://doi.org/10.36227/techrxiv.20134382.v1>
- [7] Bhattacharjee D. Generalization of grothendieck duality over serre duality in deg_n cohen-macaulay schemes representing Calabi–Yau 3-fold on Bogomolov–Tian–Todorov Theorem. Research Square; 2022u.
Available:<https://doi.org/10.21203/rs.3.rs-1781474/v1>
- [8] Seppala M. Geometry of riemann surfaces and teichmuller spaces. North Holland; 2012.
- [9] Bhattacharjee D. Generalized poincaré conjecture via alexander trick over c-isomorphism extension to h-cobordism on inclusion maps with associated kan-complex. Research Square; 2022x.
Available:<https://doi.org/10.21203/rs.3.rs-1830184/v1>
- [10] Ahmedov A, Sarsenbi A. Eigenfunction expansions of the Laplace-Beltrami Operator. LAP LAMBERT Academic Publishing; 2019.
- [11] Bujalance E, Costa AF, Martínez E. Topics on Riemann Surfaces and Fuchsian Groups (London Mathematical Society Lecture Note Series, (London Mathematical Society Lecture Note Series, Series Number 287) (1st ed.). Cambridge University Press. 2001;287.
- [12] Bhattacharjee D. Rigorously computed enumerative norms as prescribed through quantum cohomological connectivity over gromov – Witten Invariants. TechRxiv; 2022ah.
Available:<https://doi.org/10.36227/techrxiv.19524214.v1>
- [13] Bhattacharjee D, Singha Roy S, Sadhu R. Homotopy group of spheres, hopf fibrations and villarceau circles. EPRA International Journal of Research & Development (IJRD), 2022;57–64.
Available:<https://doi.org/10.36713/epra11212>
- [14] Miller H. Handbook of homotopy theory (CRC Press/Chapman and Hall Handbooks in Mathematics Series) (1st ed.). Chapman and Hall/CRC; 2019.
- [15] Bhattacharjee D. Uniqueness in poincaré-birkhoff-witt theorem over algebraic equivalence. Authorea Preprint; 2022ar.
Available:<https://doi.org/10.22541/au.165511635.53854231/v1>
- [16] Pal PB. A physicist's introduction to algebraic structures: Vector spaces, groups, topological spaces and more (Illustrated ed.). Cambridge University Press; 2022.
- [17] Chevalley C. Theory of lie groups (Dover Books on Mathematics) (Unabridged ed.). Dover Publications; 2018.
- [18] Warner S. Abstract algebra for beginners: A rigorous introduction to groups, rings, fields, vector spaces, modules, substructures, homomorphisms, quotients, . . . Group Actions, Polynomials, and Galois Theory. Get 800; 2019.

- [19] Agard S. Mostow rigidity on the line: A survey; 1988.
Available:https://doi.org/10.1007/978-1-4613-9611-6_1
- [20] Munkres, J. (2021). TOPOLOGY UPDATED. Second edition (1 January 2021); Pearson Education.
- [21] Benkhalifa M. On the classification problem of the quasi-isomorphism classes of free chain algebras. Journal of Pure and Applied Algebra. 2007;210(2):343–362.
Available:<https://doi.org/10.1016/j.jpaa.2006.09.012>
- [22] Spínola RE, Fernández-León A. CAT(k)-spaces, weak convergence and fixed points. Journal of Mathematical Analysis and Applications. 2009;353(1).
Available:<https://doi.org/10.1016/j.jmaa.2008.12.015>
- [23] Bhattacharjee D. Atiyah – Hirzebruch spectral sequence on reconciled twisted K – Theory over S – Duality on Type – II Superstrings. Authorea; 2022f.
Available:<https://doi.org/10.22541/au.165212310.01626852/v1>

© 2022 Bhattacharjee; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:

The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)

<https://www.sdiarticle5.com/review-history/92728>